QCD on the lattice - an introduction
Lecture 5

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Baryon correlation functions

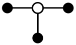
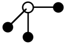
- Baryons have two-point function representations too. The simplest creation operator would be

$$\Phi(\underline{x}, t) = \Gamma_{\alpha\beta\gamma}\epsilon_{ijk} u_i^{\alpha}(\underline{x}, t) u_j^{\beta}(\underline{x}, t) d_k^{\gamma}(\underline{x}, t)$$

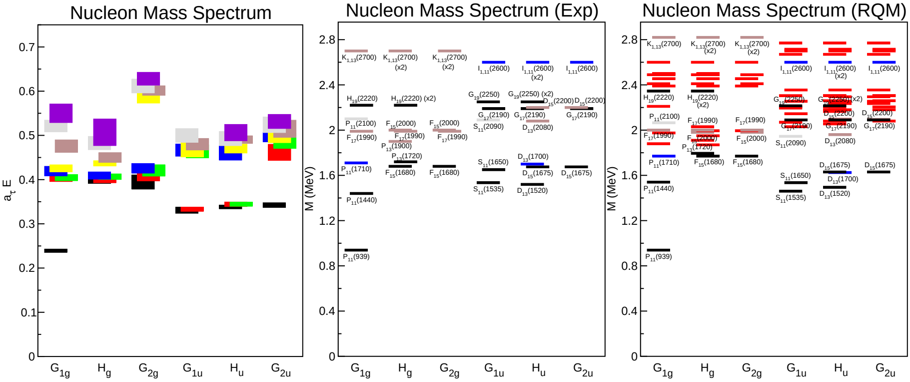
  Γ is the spin contraction to make irreducible representations of the double cover of O_h.
- There are six double cover representations: $G_1^{u,g}\, G_2^{u,g}\, H^{u,g}$
- More complicated operators can be constructed again using gluon links to make gauge invariants.

Baryon correlation functions

- Example from Phys.Rev.D72:094506,2005.

Operator type	Displacement indices
 single-site	$i = j = k = 0$
 singly-displaced	$i = j = 0,\ k \neq 0$
 doubly-displaced-I	$i = 0,\ j = -k,\ k \neq 0$
 doubly-displaced-L	$i = 0,\ \lvert j\rvert \neq \lvert k\rvert,\ jk \neq 0$
 triply-displaced-T	$i = -j,\ \lvert j\rvert \neq \lvert k\rvert,\ jk \neq 0$
 triply-displaced-O	$\lvert i\rvert \neq \lvert j\rvert \neq \lvert k\rvert,\ ijk \neq 0$

Baryon spectroscopy



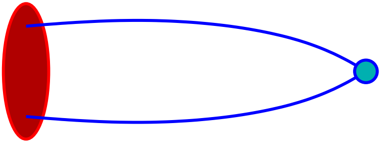
From thesis of Adam Lichtl (CMU)

Matrix elements

- Physics including the interface of QCD with the other fields in the standard model can be investigated on the lattice.
- Examples:
  1. The pion decay constant, f_π
  2. The B-meson decay constant, f_B
  3. Bag parameters B_K
  4. Form factors for electromagnetic probes interacting with mesons and baryons
- These require the evaluation of QCD matrix elements, and usually involve renormalisation to link to the other SM fields.
- Perturbation theory converges if the matching scale is high and there is a lot of work on non-perturbative matching methods.

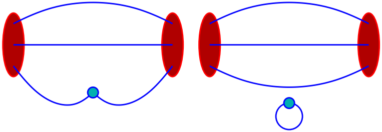
Matrix elements

- f_π requires the evaluation of a two-point function:



- Here, the blue dot represents the insertion of the axial vector current operator. Linking with the rest of the SM requires renormalisation/matching.

- Form factors require the evaluation of diagrams like:



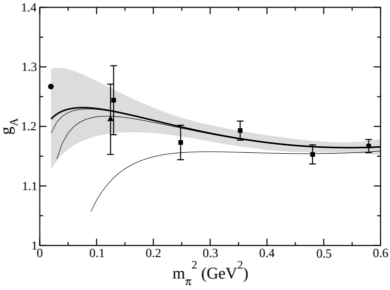
Example of current matrix-element calculation

- The axial vector $A_\mu = \bar{q}\gamma_\mu\gamma_5\tau q$ form factor of the nucleon (whose size governs β decay) is

$$\langle N(p+q,t')|A_\mu(q)|N(p,t)\rangle =$$
$$\bar{u}(p+q,t')\frac{\tau}{2}\left[g_A(q^2)\gamma_\mu\gamma_5 + g_P(q^2)q_\mu\gamma_5\right]u(p,t)$$

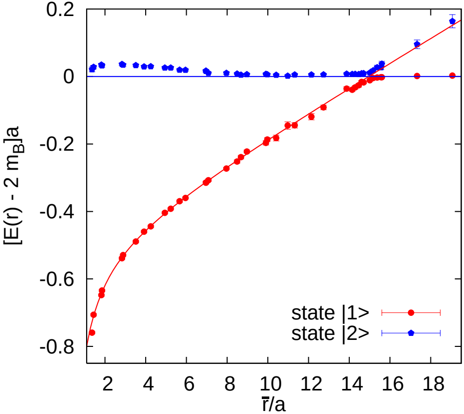
LHPC collaboration - Phys.Rev.Lett. 96 (2006) 052001

- Nucleon axial charge $g_A(0)$ vs pion mass. Experimental result is the black circle (left).

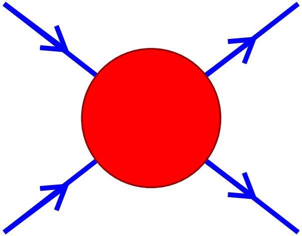


Decay physics: breaking the QCD string

- With fundamental representation quarks, the gluon string that generates the confining potential can be broken by vacuum creation of a quark anti-quark pair
- This physics is missing in the Yang-Mills theory, but present in QCD.
- This has been observed on the lattice. Phys.Rev.D71:114513,2005.



The bad news - Maiani-Testa No-go theorem



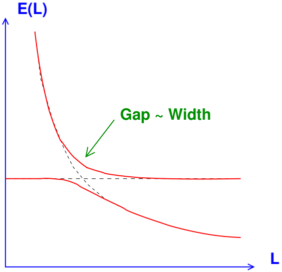
- Importance sampling Monte Carlo simulation only works efficiently for a path integral with a positive definite probability measure: Euclidean space.
- Maiani-Testa: Scattering matrix elements cannot be extracted from infinite-volume Euclidean-space correlation functions (except at threshold).
- Can the lattice tell us anything about low-energy scattering or states above thresholds?

Scattering lengths indirectly: Lüscher's method

- Scattering lengths can be inferred indirectly given the right measurements in Euclidean field theory.
- In a three-dimensional box with finite size L, the spectrum of low-lying states is discrete, even above thresholds (since the momenta of daughter mesons are quantised).
- Precise data on the dependence of the energy spectrum on L can be used to compute low-energy scattering (below inelastic threshold).
- This requires measuring energies of multi-hadron states.

Resonance energies and widths

- Above inelastic threshold, even less is known precisely.
- Resonant states will appear as "avoided level crossings" in the spectrum.
- Width can be inferred from the gap at the point where the energy levels get closest.



- Example: two states, $|\phi\rangle$ and $|\chi(p)\chi(-p)\rangle$ with $p = 2\pi/L$.

Resonance energies and widths

- Modelling these level crossings can be used to predict the energy and width of the resonance. Extracting these parameters from Monte Carlo data will require a precise scan of the energy of many states (ground-state, first excited, second . . .) in a given symmetry channel to be carried out at a number of lattice volumes.

Requirements for measuring decay widths in QCD

- Light, dynamical quarks (to ensure unitarity)
- Accurate spectroscopy in appropriate channels
- Simulations in multiple box sizes and shapes
- Access to excited states in these channels
- Ability to create multi-hadron states

$\longrightarrow$ "Next-generation" lattice spectroscopy.

Conclusions:

- **Good news:**
  1. The lattice defines field theory without reference to perturbative expansions.
  2. The lattice regulates quantum fields
  3. The lattice provides a framework for Monte Carlo simulations on (large) computers.
  4. Successes include: spectrum, QCD coupling, quark mass determinations, matrix element determinations for CKM physics
- **Bad news:**
  1. The chiral symmetry of quarks is hard to handle.
  2. For most of this to work, need the Wick rotation, so ...
  3. Studying scattering is very complicated
  4. Studying strong decay is very complicated
  5. Still can't run simulations at physical quark masses so need to extrapolate in quark masses.
- There are plenty of other challenges: QCD at finite density, ...